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The triangle
https://xa.yimg.com/kq/groups/386540/1852736325/name/quadration.pdf

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Chapter 5

The Pavillet tetrahedron

Axel Pavillet [117] quite recently discovered that if you erect perpendiculars AA_o, BB_o, CC_o (vertical dashed lines in Figure 5.1) to the plane of the triangle ABC, at A, B, C, of respective lengths $s-a$, $s-b$, $s-c$, where a, b, c are the edge-lengths of the triangle and $s=\frac{1}{2}(a+b+c)$ its semiperimeter, then the three points A_o, B_o, C_o, together with the incentre, I_o, of ABC, form an orthocentric tetrahedron. Note that $s-a$, $s-b$, $s-c$, are the lengths of the tangents from A, B, C to the incircle.

This tetrahedron has many interesting properties which will hopefully have been revealed [117] by the time this paper appears. For example, the projection of its orthocentre, O, onto the plane ABC is the **Gergonne point** of the triangle ABC; that is, the point of concurrence of the three joins, AX, BY, CZ of the vertices to the touchpoints X, Y, Z, of the incircle with the opposite edges.

In fact the joins XA_o, YB_o, ZC_o, of these points of contact to the relevant vertices of the Pavillet tetrahedron are three of its altitudes, concurring in its orthocentre, O. The plane $A_oB_oC_o$ meets the edges BC, CA, AB respectively in the points X', Y', Z' which are the harmonic conjugates of X, Y, Z with respect to BC, CA, AB. The line $X'Y'Z'$ is the polar of the Gergonne point, $\mathbb{G}_o$, with respect to the triangle ABC, and is called the **Gergonne line**. This is perpendicular to the projection onto the plane ABC of the fourth altitude of the tetrahedron, which has been called the **Soddy line** [113] of the triangle ABC, though both Apollonius and Descartes might wish to claim priority. This last line contains, in addition to the the incentre and the Gergonne point, the interior and exterior Soddy centres, S and S', and it intersects the Euler line of the triangle ABC in the **deLongchamps point** [148], D, which is the reflexion of the orthocentre of the triangle in its circumcentre, or, alternatively, the orthocentre of the triangle formed by parallels to BC, CA, AB passing respectively through A, B, C.

The so-called Soddy centres are the centres of the circles which are tangent to each of the